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Technical Notes

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Field Versus Laboratory Turbulent Boundary Layers

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Introduction

IT is difficult to overstate the technological importance of the turbulent wall-bounded flow. A vast amount of energy is spent in overcoming the turbulence skin-friction drag in pipelines and on air, water, and land vehicles. For blunt bodies, e.g., trucks and trains, the pressure drag resulting from boundary-layer separation can be several orders of magnitude higher than the skin friction, and even more energy is wasted. Heat transfer and mixing processes crucially depend on the turbulent transport for their efficient attainment. The Reynolds numbers encountered in many practical situations are typically orders of magnitude higher than those studied computationally or even experimentally (Fig. 1). High-Reynolds number research facilities are expensive to build and operate, and the few existing are heavily scheduled with mostly developmental work. For wind tunnels, additional complications due to compressibility effects are introduced at high speeds. For field conditions, say those for the boundary layers developing around commercial or military aircraft, the key question is then what are the Reynolds number effects on the mean and statistical turbulence quantities and on the organized motions of turbulence?

One of the fundamental tenets of boundary-layer research is the idea that any turbulence quantity measured at different facilities and at different Reynolds numbers will collapse to a single universal profile when nondimensionalized using the proper length and velocity scales (different scales are used near the wall and away from it). This is termed self-similarity or self-preservation and allows convenient extrapolation from the low-Reynolds number laboratory experiments to the much higher Reynolds number situations encountered in typical field applications. The universal logarithmic profile that describes the mean streamwise velocity in the inner layer of any wall-bounded flow is the best known example of the stated classical idea. In the present research, data from numerous numerical and physical experiments have been carefully analyzed, and it is shown that boundary layers, in fact, never achieve true self-preservation and that neither inner nor outer scaling is strictly valid. Reference 1 documents the details of our findings. The purpose of the present Technical Note is to present an informative summary of value for the readers of the *AIAA Journal*.

The implications of this result are far reaching. Resolution of the full equations, via direct numerical simulations, at all but the most modest values of Reynolds number is beyond the reach of current or near-future computer capabilities. Modeling will, therefore, continue to play a vital role in the computations of practical flows using the Reynolds-averaged Navier–Stokes equations. Flow modelers, in attempting to provide concrete information for the designers of ships, submarines, and aircraft, heavily rely on similarity principles in order to model the turbulence quantities and go around the well-known closure problem. Since practically all turbulence models are calibrated to reproduce the law of the wall in simple flows, failure of this universal law virtually guarantees that Reynolds-averaged turbulence models would fail too. Developers of flow control devices often have to extrapolate the widely available low-speed results to high-speed flows of practical interest where no data are available. For scientists and engineers the message is essentially back to the drawing board!

Canonical Wall-Bounded Flow

Consider the simplest possible turbulent wall-bounded flow, that over a smooth flat-plate at zero incidence to a uniform, incompressible flow or its close cousin, the two-dimensional, fully-developed channel flow. Leaving aside for a moment the fact that such idealized flow does not exist in practice, where three-dimensional, roughness, pressure-gradient, curvature, wall compliance, heat transfer, compressibility, stratification and other effects might be present individually or collectively, the canonical problem itself is not well understood. Most disturbing from a practical point of view are the unknown effects of Reynolds number on the mean flow, the higher order statistical quantities, and the flow structure.

The subject is broad, and here it is discussed mostly in light of four questions.[2] One of the earliest studies of the Reynolds number effect in turbulent boundary layers was due to Coles.[3,4] When measurements of mean-velocity profiles were expressed in inner-layer form based on directly measured local friction values, a logarithmic region was found to exist even at a Reynolds number based on momentum thickness and freestream velocity of $Re_\theta = 50 \times 10^3$. The wall-layer variables appear to describe the mean flow in the inner layer universally in flat plates, pipes, and channels at all Reynolds numbers, although even this widely accepted conclusion has recently been critically questioned.[1,5] George et al.[5] maintain that the growing, inhomogeneous boundary layer is governed by a power law, with explicit Reynolds number dependence, and is different from the fully developed channel flow which is homogeneous in the streamwise direction and is governed by a logarithmic law.

In a boundary layer, the behavior of the outer layer, when expressed in terms of wall-layer variables by the strength of the wake component ΔU^+, which is the maximum deviation of the mean-velocity profile from the log law, appeared to reach an asymptotic value for $Re_\theta > 6 \times 10^3$. Above this limit, the inner- and outer-layer mean flows are expected to reach an asymptotic state which the turbulence quantities are also hypothesized to follow. This is, however, not the case since the wake component starts decreasing, albeit slowly, at about $Re_\theta > 15 \times 10^3$. This raises the question, does the mean flow ever achieve true self-preservation?

The situation is murkier for higher order statistics. Measurements in pipes,[6] channels,[7] and boundary layers[8–12] are beginning to show that the turbulence quantities do not scale with wall-layer variables even in the inner layer. Therefore, the question arises, can we apply the mean-flow scales to turbulence?

Apart from that in turbulence modeling, knowledge of the Reynolds number effects is useful to flow control. This is because experimental investigations at low-Reynolds numbers, i.e.,

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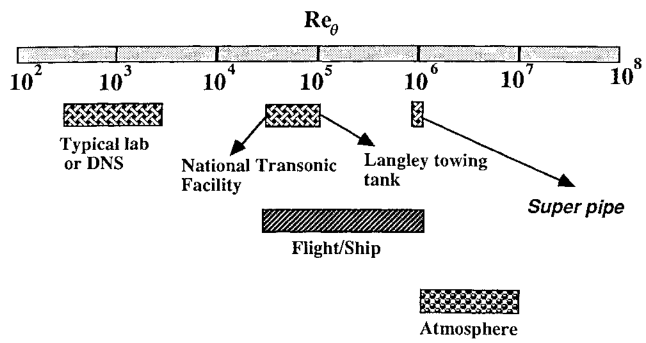

Fig. 1 Ranges of momentum-thickness Reynolds number for different facilities and for field conditions.

lower speeds and/or smaller length scales, are less expensive. Most flow control devices are, therefore, developed and tested at rather low speeds. Extrapolation to field conditions is not always straightforward though, and it often comes to grief. The outer-layer-device drag reduction experiments of Anders[13] show that above the well-known Reynolds number limit of $Re_\theta > 6 \times 10^3$, the maximum skin-friction reduction and the recovery length (the latter with some exception) do not remain constant but reduce with increasing Reynolds number. The loss of performance at higher Reynolds numbers is puzzling, and Anders attributed it to a significant change in the turbulence structure. In this background, the question, does the turbulence structure change when $Re_\theta > 6 \times 10^3$, is discussed.

Consider another puzzling high-Reynolds number behavior. In the 1950s, Clauser[14] had experimentally shown that in a turbulent boundary layer at a given Reynolds number, disturbances survive much longer in the outer layer than in the inner layer. He demonstrated this by placing a circular rod in the outer and inner layers of a fully developed wall layer. In viscous drag reduction techniques where a device drag penalty is involved a recovery length of $\mathcal{O}[100\delta]$ is desirable to achieve a net gain.[15] To date, with outer-layer devices, such recovery lengths have been achieved only at low-Reynolds numbers as mentioned earlier. One normally expects the recovery length to be far less if the disturbances are applied near the wall, and the length to reduce even more as Re_θ is increased. However, the data of Klebanoff and Diehl[16] are re-examined in Ref. 1 and show that, in fact, at higher Reynolds numbers, an opposite trend sometimes takes place. This unexpected result indicates a serious difficulty in the extrapolation of low-Reynolds number results. The fourth question is concerned with this aspect of the Reynolds number effect.

Two-Scale Problem

An inspection of the distribution of viscous and turbulence shear stresses in a typical wall-bounded flow demonstrates the presence of three distinct regions (Fig. 2). The time-averaged viscous stress is important only near the wall. This wall layer (viscous sublayer plus buffer layer) is followed by a region of approximately constant Reynolds stress (overlap layer). Finally, a wake region is characterized by a diminishing turbulence shear stress, reaching zero at the edge of the boundary layer.

Unlike the second and third regimes, the extent of the first region, expressed in wall units, does not depend on Reynolds number. As a percentage of the boundary-layer thickness, however, the viscous region continuously shrinks as the Reynolds number increases, as shown in Fig. 2 for $Re_\theta = 10^3$ (typical of laboratory experiments) and $Re_\theta = 10^5$ (typical of field conditions). The upper extent of the overlap region is a constant fraction of the boundary-layer thickness, but varies with Reynolds number when referenced to the viscous length scale (equal to approximately 100 wall units at $Re_\theta = 10^3$ and 5500 wall units at $Re_\theta = 10^5$). Except for the dissenting viewpoint mentioned earlier, most of the available evidence appears to indicate that both the viscous region and the constant-Reynolds stress region are similar in all wall-bounded flows. In contrast, the outer layer is different in internal flows and boundary layers. Profiles of the mean velocity and other turbulence statistics can be constructed from scaling considerations of the three distinct regimes.

Attention is drawn in Ref. 1 to some aspects of the emerging description of the structure of high-Reynolds number turbulent boundary layers. Both the inner- and outer-layer structures are affected by Reynolds number.[17,18] The turbulence quantities do not accurately scale with wall-layer variables in the inner layer. As aptly illustrated by Kailasnath,[19] the classical similarity theory of wall-bounded flows that asserts a universal description for the near-wall flow is found to be increasingly deficient as the questions become more detailed. The boundary layer might continue to change indefinitely as the inner and outer scales are forever disparting. The outer-layer turbulence structure, as inferred from measurements of the skewness factor of the streamwise velocity fluctuations, peak shear-correlation-coefficient, intermittency, u-v quadrant distributions, and streamwise scales, is greatly changed at extremely high-Reynolds numbers[20,21] and new structures probably evolve due to vortex-vortex interactions.

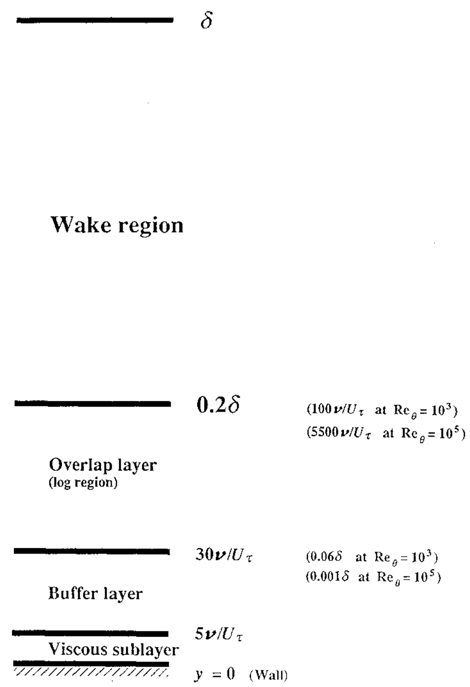

Fig. 2 Schematic of the different regions within a wall-bounded flow at typical low- and high-Reynolds numbers; Re_θ is the momentum-thickness Reynolds number, y the surface-normal coordinate, δ the boundary-layer thickness, and ν/U_τ the viscous length scale (kinematic viscosity divided by friction velocity).

The numerically simulated low-Reynolds number, flat-plate turbulent boundary layers are characterized by a paucity of scale and a lack of vortex-vortex interaction. Studies of the very low-Reynolds number turbulent boundary-layer structure might not inherently involve several aspects of the high-Reynolds number structure which may be crucial to flow control through turbulence manipulation.

Why does the mean flow scale, at least approximately, with wall-layer variables in the inner layer yet turbulence quantities do not? At a relatively low-Reynolds number, say $Re_\theta = 500$, the inner-layer mean flow appears to be already universal (as mentioned earlier, even this seemingly infallible result has been seriously challenged[5]); so why should not the low-Re_θ structure be universal? These questions erroneously imply that there is a first-order direct connection between the mean flow and turbulence in a wall-bounded flow as in a free-shear flow. In a two-dimensional mixing layer, for example, the experimentally observed two-dimensional rollers are the direct result of an inviscid instability of the mean-velocity profile. Their characteristic dimension is equal to the layer thickness, and they contain almost all of the mean-flow vorticity. In a wall-bounded flow, on the other hand, the omnipresent hairpin vortices are the result of a secondary or a tertiary instability, and their diameters are typically much smaller than the boundary-layer thickness. The three-dimensional hairpins contain only a portion of the mean flow vorticity—that is, they are farther removed from the two-dimensional mean flow.

Experience with turbulence modeling also suggests that the turbulence in a wall-bounded flow is not derived directly from the mean flow. In the earliest turbulence models, shear stress was derived from the mean-velocity profile. Such models have not been widely successful. Townsend[22] and Bradshaw et al.[23] have argued that instead there is a much closer connection between the shear stress and the turbulence structure. Townsend's work was limited to the near-wall region, whereas Bradshaw et al.[23] have extended the argument to the entire shear layer. Direct measurements of the so-called typical eddies have supported their assertion.[24,25]

Summary

The conclusions for Reynolds number effects as detailed in Ref. 1 may be summarized as follows.

1) The widely accepted asymptotic state of the wake component is present only in the range $6 \times 10^3 < Re_\theta < 1.5 \times 10^4$. At higher values, it drops although at a much slower rate.

2) The Clauser's shape parameter is strongly Reynolds number dependent at $Re_\theta < 10^3$ and weakly dependent above that.

3) Alternatives to the logarithmic mean-velocity profile have been periodically proposed. Such heretical ideas deserve further scrutiny. Independent confirmation via well-controlled experiments that cover a wide range of Reynolds numbers, resolve the linear region, and directly measure the wall-shear stress is needed.

4) The freestream turbulence effect is dependent on Reynolds number.

5) Turbulence measurements with probe lengths greater than the viscous sublayer thickness (~5 wall units) appear to be unreliable, particularly near the wall.

6) Unlike the mean flow, the statistical turbulence quantities do not scale accurately with the wall-layer variables over the entire inner layer. Such scaling applies over only a very small portion of the inner layer adjacent to the wall.

7) At low-Reynolds numbers, the peak u-turbulence intensity increases slightly with Reynolds number in both channels and flat plates.

8) The distance from the wall where the streamwise turbulence intensity peaks appears to scale with inner variables.

9) In contrast, the corresponding distances, expressed in wall units, for both the normal fluctuations and the Reynolds stress move away from the wall as the Reynolds number increases. At high Re, the peak normal turbulence intensity and the peak Reynolds stress occur substantially outside the viscous region.

10) The wall-pressure rms increases slightly with Reynolds number.

11) Systematic changes in the mean and higher order statistics as the Reynolds number varies could be considered as proper first-order trends within the framework of an asymptotic theory. At finite Reynolds numbers, the additive composite expansion formed from the inner and outer expansions of any turbulence quantity provides the only uniformly valid approximation in the matched region.

12) In flat plates, trip memory can survive the statistical turbulence quantities at even $Re_\theta > 6 \times 10^3$, where the mean flow is said to have reached an asymptotic state.

13) The Reynolds number dependence of the post-transition relaxation length of both the mean and turbulence quantities is not well understood.

14) In pipe flows, the wave nature of the viscous sublayer, which is observable at low-Reynolds numbers, gives way to a poorly understood random process at high-Reynolds numbers.

15) Although the variously defined (small) length scales differ greatly from each other at low-Reynolds numbers, they all asymptote to the mixing length at much higher Reynolds numbers ($Re_\theta > 1.0 \times 10^4$).

16) The outer-layer structure changes continuously with Reynolds numbers, and very little is known about the structure of very high-Reynolds number turbulent boundary layers.

17) The aspect ratio of the omnipresent hairpin vortices increases with Reynolds number as they also become skinnier. In a large structure, the number of constituent hairpin vortices per unit wall area increases with Reynolds number.

18) Changes in the wall-bounded flow physics could be described as due to changing the scale ratio, $\delta^+ \equiv \delta U_\tau/\nu$, and not the Reynolds number per se. In a given boundary layer, δ^+ changes downstream at a rate slightly lower than Re_θ. The influence of the wall changes from nonlocal to local as this scale ratio increases.

19) There is a dire need for high-resolution, reliable measurements of mean and statistical turbulence moments at high-Reynolds numbers in smooth, flat-plate turbulent boundary layers.

20) Reynolds number effects in canonical flows cannot always be extrapolated to noncanonical cases in a simple straightforward manner.

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Weighted Least Squares Method of Grid Generation

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Introduction

ONE of the important steps required to accurately solve a problem numerically is the generating of appropriate grids. To the authors' knowledge, the transfinite interpolation method[1,2] is the fastest of existing grid generation methods. However, if the resulting grid system cannot fulfill the users' requirement, one must redistribute the grid points on the boundaries or redefine the blending functions, which might take a long time for a complex domain. In this study, the authors propose the creation of a grid smoother to improve the local grid distortion, nonsmoothness, and overlapping of the transfinite interpolated grid points. The grid smoother, based on the weighted least square method,[3] is designed to cover either regular or irregular subregion(s) on the computational domain and to maintain the grid clustering or stretching characteristics of the original grid distribution.

Among the existing adaptive grid methods, the multiple one-dimensional adaptive grid method[4,5] is one of the fastest methods. Unfortunately, this method has the drawback of excessive grid distortion, even if the Jeng and Liou smoothing version[5] is applied. Therefore, the second purpose of this study is to smooth the resulting grid finding through the application of the Jeng and Liou modified multiple one-dimensional adaptive grid method.

Formulation

Suppose that we have an initial grid system, $(x^0_{i,j}, y^0_{i,j})$, which has some grid imperfections. First, define the two-dimensional grid functional to be

$$F = \sum_{i,j} q_{i,j} \left\{ \sum_{k=\pm 1} \left[\frac{(x_{i+k,j} - x_{i,j})^2 + (y_{i+k,j} - y_{i,j})^2}{\sqrt{\left(x^0_{i+k,j} - x^0_{i,j}\right)^2 + \left(y^0_{i+k,j} - y^0_{i,j}\right)^2}} + \frac{(x_{i,j+k} - x_{i,j})^2 + (y_{i,j+k} - y_{i,j})^2}{\sqrt{\left(x^0_{i,j+k} - x^0_{i,j}\right)^2 + \left(y^0_{i,j+k} - y^0_{i,j}\right)^2}} \right] + 2\nu \left[\frac{(x_{i+1,j} - 2x_{i,j} + x_{i-1,j})^2 + (y_{i+1,j} - 2y_{i,j} + y_{i-1,j})^2}{\sum_{k=\pm 1} \sqrt{\left(x^0_{i+k,j} - x^0_{i,j}\right)^2 + \left(y^0_{i+k,j} - y^0_{i,j}\right)^2}} + \frac{(x_{i,j+1} - 2x_{i,j} + x_{i,j-1})^2 + (y_{i,j+1} - 2y_{i,j} + y_{i,j-1})^2}{\sum_{k=\pm 1} \sqrt{\left(x^0_{i,j+k} - x^0_{i,j}\right)^2 + \left(y^0_{i,j+k} - y^0_{i,j}\right)^2}} \right] \right\} \tag{1}$$

where the denominators serve as weighting functions, $q_{i,j}$ denotes the localized switching function, and ν is a user specified parameter. The weighting function takes the first power of the grid spacing to ensure grid smoothness in case the initial grids are unequally spaced. By taking the first-order partial derivatives of F with respect to $x_{i,j}$ and $y_{i,j}$ separately, and by setting these derivatives to be zero, the linear grid equations for the x coordinate are easily formulated.

$$\begin{aligned} &a_2 x_{i+2,j} + a_1 x_{i+1,j} + a_0 x_{i,j} + a_{-1,j} x_{i-1,j} + a_{-2} x_{i-2,j} \\ &\quad + b_2 x_{i,j+2} + b_1 x_{i,j+1} + b_{-1} x_{i,j-1} + b_{-2} x_{i,j-1} = 0 \end{aligned} \tag{2}$$

where the coefficients a and b are functions of the initial coordinates ($x^0_{i,j}$ and $y^0_{i,j}$) and equations for the y coordinate are in a similar form with x replaced by y. Note that in these final grid equations, the terms of the grid spacings of Eq. (1) correspond conceptually to the terms $x_{\xi\xi}, x_{\eta\eta}, \ldots$, of the elliptic equation method, whereas terms of the second-order differencing conceptually correspond to the fourth order smoothing terms $x_{\xi\xi\xi\xi}, \ldots$.

If $q_{i,j}$ of Eq. (1) is replaced by the unit value, the method can be applied to the entire grid. In general, the transfinite interpolation method generates smooth grid points over the whole computational domain except at some isolated regions. Therefore, it is reasonable to apply the method of Eqs. (1) and (2) only at points around these isolated regions by properly choosing $q_{i,j}$. The switching functions can be defined in different ways. For example, if grid overlapping is to be avoided, they are defined as

$$q_{i,j} = \text{integer}\left| \frac{|J_{i,j}| - J_{i,j} + 2\epsilon}{2|J_{i,j}| + \epsilon} \right| \tag{3}$$

where ϵ is a small positive parameter which prevents dividing by zero, and $J = x_\xi y_\eta - x_\eta y_\xi$. Equation (3) returns a zero value of $q_{i,j}$ for $J > 0$ and an unit value for $J \leq 0$.

To obtain a smooth transition between the adjusting region and that without adjusting, every adjusting region where $q = 1$ should contain a transition margin enclosing the unsatisfactory grid points. Therefore, the following extension is performed twice:

$$\begin{aligned} t_{i,j} &= q_{i-1,j-1} + q_{i,j-1} + q_{i+1,j-1} + q_{i-1,j} + q_{i,j} + q_{i+1,j} \\ &\quad + q_{i-1,j+1} + q_{i,j+1} + q_{i+1,j+1} \end{aligned} \tag{4}$$

where $q = 0$ or 1, and then

$$q_{i,j} = \text{integer}\left| \frac{s_{i,j} + 1}{2} \right|, \qquad s_{i,j} = \text{Sign}(1., t_{i,j} - 0.5) \tag{5}$$

where the Sign symbol denotes a Fortran function such that $s_{i,j} = 1$ if $t_{i,j} \geq 1$ and otherwise equals 0. If the resulting grid distribution is not satisfactory, the extension can be repeatedly performed. By substituting these switching functions into Eq. (1) and solving Eq. (1) in terms of the point Gauss-Seidel iteration, the present method can be applied to every arbitrary subregion of the computational domain.

Results and Discussions

The linear transfinite interpolation method,[2] whose independent variables are ξ and η, is first applied to a simple geometry to construct an algebraic grid system (20 × 20 points). To demonstrate the characteristics of the initial grid system, assume that grid clustering is desired at the center region. Since the grid point adjustment along the boundaries is purposly made improper, grid overlapping is present at the center region as shown in Fig. 1. After employing the present local adjusting method to improve the undesired grid overlapping and to maintain the grid clustering, the result is satisfactory, as shown in Fig. 2. The CPU time required is 6.4 s on an HP720 workstation, which is significantly less than the 18.8 s required if the entire grid is adjusted.

The second example is an inviscid transonic flow over a 10% bump, where the inflow Mach number takes a value of 0.675. The initial grid found by the linear transfinite interpolation method has a grid size of 61 × 17 points. Because of the space limitation, both the initial grid system and the initial solutions calculated by the Harten-Yee minmod total variation diminishing (TVD) scheme[6] are not shown. Then, the application of the Jeng and Liou[5] multiple one-dimensional adaptive grid scheme (with $\lambda_\xi = 2$ and $\lambda_\eta = 0.5$, where ξ = const is a vertical grid line) gives the grid distribution of Fig. 3. Although the multiple one-dimensional adaptive grid scheme is very

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